Geophysics class algebra/trig/SN
Evaluation worksheet

Name:_____

1. Draw a real number line and shade the following interval(s).

   (a) $[-2, \infty)$
   (b) $(-4, 1/2)$
   (c) $[-\pi/2, -1)$
   (d) $(-\infty, 732)$
   (e) $[0, 2\pi]$
   (f) $(-\infty, \infty)$

2. Simplify

   (a) $\left(\dfrac{4a^4}{b^5}\right)^2 \dfrac{b^3}{a^7}$
   (b) $\dfrac{1}{t+2} + \dfrac{t}{t-1} - \dfrac{3t}{t^2+t-2}$
   (c) $\dfrac{\frac{1}{y} + \frac{y}{y-1}}{\frac{2}{y-1} - \frac{1}{y}}$
   (d) $3(x+5)^{\frac{-1}{4}}(x-2)^2 + (x+5)^{\frac{3}{4}}(x-2)$

3. Solve the following inequalities in terms of intervals and draw the solution set on a real number line:

   (a) $1 - 7x \leq 3 + 4x$
   (b) $|-5x + 3| \leq 4$
   (c) $x^3 - x^2 < 0$
   (d) $0 < |x - 5| < 1/2$

4. Draw a single coordinate plane and plot the following points

   (a) $(3, 4)$
   (b) $(-\pi, 1)$
   (c) $(\pi, -1)$
   (d) $(-1/2, -1/4)$

5. State the formula for the distance between two points in the plane. Find the distance between the points $(2, 5)$ and $(4, -7)$.

6. Find the equation of the line through the following points.

   (a) $(0, 3)$ and $(2, 1)$
   (b) $(-1, -1)$ and $(1/2, 3)$

7. State any definition of a function $y = f(x)$. State the definition of the domain of $f(x)$.

8. State the Vertical Line Test (VLT).

9. State the definition of what it means for a function f to be increasing on an interval I. Repeat for decreasing.

10. Sketch the following graphs (at most 2 graphs in a coordinate plane, please):

    (a) $y = x$
    (b) $f(x) = x^2$
    (c) $g(x) = x^3$
    (d) $y = \dfrac{1}{x}$
    (e) $y = |x|$
    (f) $x^2 + y^2 = 4$
    (g) $y = -x/2 + 4$
    (h) $\{(x, y) \mid x^2 + y^2 \leq 3\}$
    (i) $y = (x+3)^2 - 1$

11. Let $f(x) = 3 - 4^x + x^2 - 5x + \dfrac{1}{3x} - \sqrt{x}$. Find the following, if possible:

    (a) $f(1)$
    (b) $f(0)$
    (c) $f(-1)$
    (d) $f(a)$
    (e) $f(a+h)$
    (f) $f(x+h)$

12. Let $f(x) = \sqrt{x}$. Let $g(x) = x - 19$. Find $(f \circ g)(x)$, then state its domain.

13. Let $f(x) = 1 - x^9$, $g(x) = \dfrac{1}{x}$, and $h(x) = \cos x$. Find the following compositions:

    (a) $(f \circ g)(x)$
    (b) $(h \circ f)(x)$
    (c) $(f \circ g \circ h)(x)$

14. Graph $f(x) = \begin{cases} \frac{x}{2} + 4, & \text{if } x < -2; \\ x^2, & \text{if } -2 \le x < 1; \\ x^3 - 9, & \text{if } x \ge 2. \end{cases}$

Trigonometry

15. Recall that angles can be measured in radians or degrees. (We will stick to radians.) Recall that $\frac{\text{rad}}{\pi} = \frac{\text{deg}}{180^\circ}$. Fill in the blank with either the radian or degree measure (should be clear from context).

    (a) _____ $= 0^\circ$.
    (b) $3\pi/8 =$_____.
    (c) _____ $= 275^\circ$.
    (d) $\pi/2 =$_____.

16. What are the "important" angles for trigonometry?

17. Draw a unit circle and label the "important" angles.

18. On the unit circle, if θ is an angle, which coordinate of the intersection of the terminal side of the angle with the unit circle is the $\sin\theta$? the $\cos\theta$? What's $\tan\theta$?

19. In what quadrants is $\sin\theta$ positive? Repeat for $\cos\theta$, $\tan\theta$.

20. State the definitions of $\csc\theta$, $\sec\theta$, $\cot\theta$.

21. State the right triangle definitions of the trigonometric functions. What does "SOH CAH TOA" mean?

22. Make a table in which you list the "important" angles and their values under the sine, cosine, and tangent functions.

23. Find the sine and cosine of $-\frac{7\pi}{6}$.

24. State the Pythagorean identities. (there are 3 of them)

25. Is $\sin\theta$ even/odd? Is $\cos\theta$ even/odd? (What do even and odd mean?)

26. What is the period of $\sin\theta$? $\cos\theta$? $\tan\theta$?

27. Graph two full period of $\sin\theta$, $\cos\theta$, $\tan\theta$. (on separate graphs)

28. If $\sin\beta = -1/3$ and $\pi < x < 3\pi/2$, find the remaining trigonometric ratios (functions).

29. If $\cot\beta = 3$ and $\pi < x < 2\pi$, find the remaining trigonometric ratios (functions).

30. Prove the following identities:

    (a) $\sin\theta\cot\theta = \cos\theta$
    (b) $\sec y - \cos y = \tan y \sin y$
    (c) $\cos\left(\frac{\pi}{2} - x\right) = \sin x$
    (d) $\sin^2 x - \sin^2 y = \sin(x+y)\sin(x-y)$

31. If a right triangle has a hypotenuse of length 4 in, and an angle of $\pi/3$ radians, find the length of the opposite side of the angle. Use trig functions.

32. Find all values of x in the interval $[0, 2\pi]$ that satisfy the equation.

    (a) $2\cos x - 1 = 0$
    (b) $4\sin^2 x = 1$
    (c) $\sin 2x = \cos x$
    (d) $\sin x = \tan x$